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Economics 301

Isoquants and the Producer's equilibrium

Below, we work through an example of the hiring decision facing a producer with two variable factors that must be hired to produce a specific good (let's say good X).

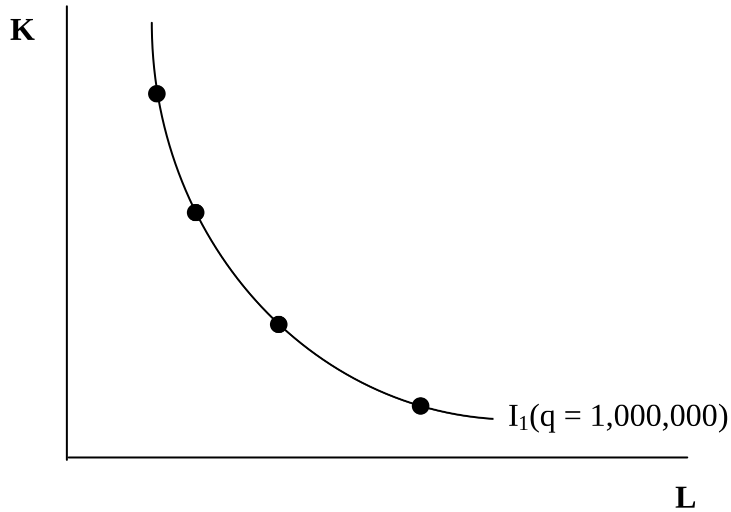
Let's begin by making some assumptions. First, rather than go through the assumptions of this model, we'll refer you to the text where those assumptions are stated. We'll assume here that the two variable factors are capital (K) and labor (L) and that our producer's production function is:

$$q = \sqrt{K \cdot L} \qquad (1)$$

Let's say that our producer chooses to produce 1000 units of good X. If q = 1000, then we know there are many possible combinations of L and K that would allow this to happen. If we take (1), set q = 1000 and solve for K, we get the following equation below:

$$K = \frac{1{,}000{,}000}{L} \qquad (1a)$$

Graphing this equation, with K on the vertical axis and L on the horizontal axis, we obtain the line I_1 on the graph below. Each point on I_1 represents a possible combination of K and L that allows the producer to get q = 1000. We call this curve an isoquant.



As we turn to deciding how many units of L and K to hire, let's focus on the cost of these factors. Let's assume that each factor was a price which does not vary with output. That is, the price of each unit of labor (w) and the price of each unit of capital (r) are assumed constant. Let's assume those prices are w = \$10 and r = \$50 (where w is the cost of each unit of labor, and r is the cost of each unit of capital). Given that these are the producers only costs, the producer's total cost would correspond with (2).

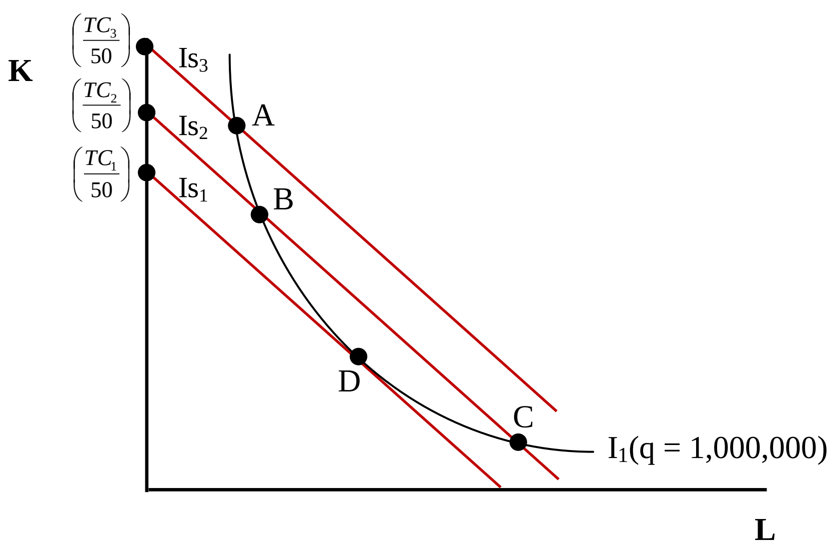
$$10L + 50K = TC \qquad (2)$$

The total cost equation in (2) is known as an isocost line and it's similar to a budget constraint. Knowing that she must produce 1000 units of good X, the producer will seek to choose the least expensive combination of L and K capable of producing those 1000 units of output. E.g., both L = 1000, K = 1000, and L = 10,000, K = 100, allow the firm to produce 1000 units, but the first combination will cost the firm \$60,000 and the second \$105,000. The firm wants to minimize TC, which puts a constraint on this decision.

Let's take isocost equation in (2) and rearrange it by solving for K as follows:

$$K = \left(-\frac{1,}{5}\right)L + \left(\frac{TC}{50}\right) \qquad (2a)$$

Note that this is the equation for a line, that (-1/5) is the slope and (TC/50) is the vertical intercept. Given that TC will vary with every combination of L and K hired, we would have a large number of potential isocost lines on our graph. The firm would be interested in finding the lowest possible isocost line that still allows the firm to operate on I_1.



Note that points A, B, C and D are each located on this firm's isoquant I_1, which means they would each allow the firm to produce 1000 units. The question is, which line is the lowest cost isocost line? Note that the vertical intercept for each of these 3 lines (Is_1, Is_2, Is_3) is included on the graph. Of course, since the only difference between these 3 intercepts is the amount of TC, we do know that even though Is_1, Is_2 and Is_3 each allow the firm to produce 1000 units, Is_1 involves a lower TC than what possible with Is_2 or Is_3. Is there another isocost line even lower than Is_1? Yes, but because those lower isocost lines would not intersect I_1, none of those lower isocost lines would give us q = 1000 since they would not intersect I_1.

Note as well that point D involves a tangency point between I_1 and Is_1. I.e., the slope of these two curves is equal at point D. The slope of the isocost line is (-1/5), which we get from (2a), but the slope of the isoquant is not constant since the isoquant is not a straight line. Therefore, we must refer to the slope of the isoquant in terms of a specific point. Rather than derive the slope mathematically, we'll simply call it by its name, the marginal rate of technical substitution, and abbreviate it as MRTS. A tangency point at point D would then imply that MRTS = -1/5.

Based on what we've established up to this stage of the discussion, it's clear that our equilibrium must involve 3 different conditions being simultaneously met.

a) We must operate on an isocost line (i.e. here it must be true that 10L + 50K = TC)

b) We must operate on a specific isoquant (i.e., here it must be true that $K = \frac{1{,}000{,}000}{L}$)

c) Our equilibrium will involve a tangency point between the isocost line and our isoquant (i.e. here it must be true that MRTS = 1/5)

In order to solve for L and K, we must have an equation for the MRTS. Although it's possible to derive that equation from (1), deriving it would involve the use of calculus and rather than march through some unnecessary math, let's just provide an equation for the MRTS that is based on the production function in (1).

As it turns out, the slope of the isoquant is the ratio of the marginal products, MP_L/MP_K, and if we derive the MRTS using this information, we would get:

$$MRTS = -\frac{K}{L} \qquad (3)$$

Therefore, a tangency between the isocost and isoquant would imply MRTS = -1/5, giving us:

$$-\frac{K}{L} = -\frac{1}{5} \qquad (3a)$$

We can solve for K* and L* now, by working with (1a), (2a) and (3a) – the three conditions that must be met at point D. Our starting point is to take (1a) and substitute that equation into both (2a) and (3a), which gives us the following two equations:

$$10L + 50\left(\frac{1{,}000{,}000}{L}\right) = TC \qquad (4)$$

$$\left(\frac{\left(1{,}000{,}000 / L \right)}{L} \right) = \frac{1}{5} \qquad (5)$$

If we solve (5) for L and substitute that result into (4), we can solve for TC (the only variable that would be remaining). Note that this value for TC is the lowest TC possible (as per our discussion above), so we'll call this value TC*. The result is:

TC* = $44,721.36

We can then substitute TC* into (4) and solve for L* (rounded to the nearest whole number).

L* = 2,236

We can solve for K* by substituting L* into (3a).

K* = 447

In summary, the lowest cost combination of L and K that allow this firm to produce 1000 units involves hiring 2,236 units of labor and 447 units of capital. In doing so, the firm will incur a total cost of $44,721.36.