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Econ 301

Substitution and Income Effects in the Indifference Curve model

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Let's assume that Homer Simpson is a typical consumer within the town of Springfield who consumes varying amounts of bacon and some composite good we'll call "all other goods". This composite good is comparable to the basket of consumer goods that we talk about when calculating the consumer price index, except that the composite good everything a typical consumer needs, other than bacon. Assume that Q_B is the quantity of bacon consumed, and that Q_A is the quantity of all other goods consumed.

Homer's utility function is given as: $U(Q_B Q_A) = \sqrt{Q_B \cdot Q_A}$

We can transform this utility function into an equation for a specific indifference curve (set the function equal to a specific amount of utility, and solve for Q_A or Q_B. Of course, the variable we solve for when creating an equation for one specific indifference curve determines which quantity goes on which axis, so let's assign a variable to each axis on our (eventual) graph. Let's assume Q_A is on the vertical axis, and Q_B is on the horizontal axis. That said, we can now provide an equation for the marginal rate of substitution (i.e. the slope of Homer's indifference curve) between bacon and all other goods (under the assumption that Q_A is on the vertical axis):

$$MRS_{RB} = -\frac{Q_A}{Q_B} \qquad (1)$$

Let's assume further that the price of bacon is \$2, the price of all other goods is \$10, and that Homer's income is \$1000. This information gives us Homer's budget constraint:

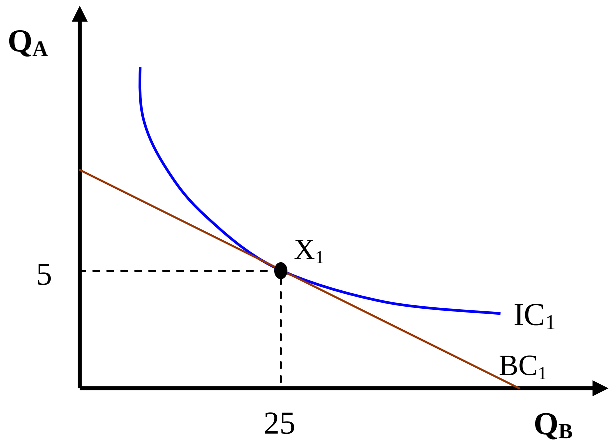
$$10Q_A + 2Q_B = 100 \qquad (2)$$

Rearranging (2) so that we can include the budget constraint on our graph, we have:

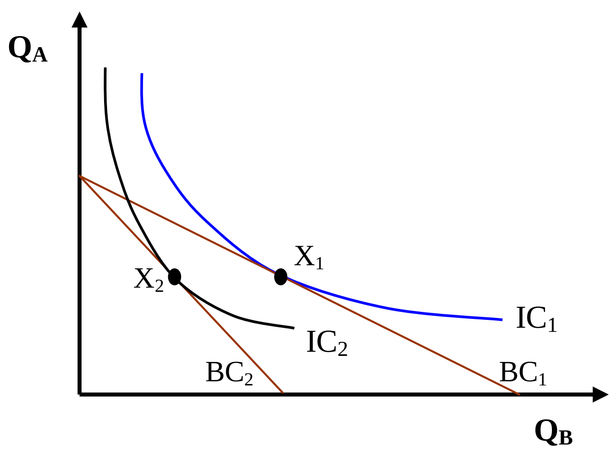
$$Q_A = -0.2Q_B + 100 \qquad (2a)$$

We can take (1) and (2a), and use some basic algebra to determine that Homer maximizes his utility by purchasing $Q_A* = 5$ and $Q_B* = 25$. Using Homer's utility function, we know that this bundle allows him to achieve $5\sqrt{5}$ units of utility (i.e. utils).

Homer's consumer equilibrium occurs below at point X_1, where the (blue) indifference curve IC_1 is tangent to the (red) budget constraint BC_1.



How is the graph above affected when the price of bacon increases from \$2 to \$4? This change is shown on the graph below. The budget constraint becomes steeper (shifts from BC_1 to BC_2) and Homer moves to a new (black) indifference curve IC_2 and a lower level of utility at pt. X_2. If we calculate the new consumer equilibrium at pt. X_2, we get $Q_A* = 5$ and $Q_B* = 12.5$.

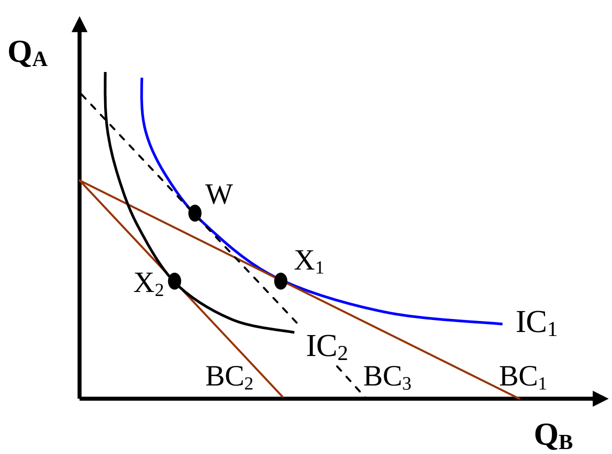


Notice, however, that the price change included two actions. First, the movement from X_1 to X_2 involved a change in the slope of the budget constraint (i.e. the slope of BC_2 and IC_2 at X_2 on the new indifference curve is not the same as the slope of BC_1 and IC_1 at X_1). This implies a change in what we call relative prices. When the price of bacon increases relative to all other goods, consumers will not view bacon quite the same after this price change. When the relative price of something goes up (as it did with bacon), we know that consumers decrease their purchase of the (relatively) more expensive good and increase their purchase of the (relatively) less expensive good – i.e., they substitute away from the relatively more expensive good. Note also that this price change causes Homer to experience lower purchasing power as well. In a macroeconomics course, we talk about purchasing power as the income you need to buy a specific amount of goods (also called "real income). A change in purchasing power is equivalent to a change in income. These two actions form the analytical basis for what we call the substitution effect and the income effect.

Substitution and Income Effects

When prices rise, consumers lose purchasing power, but let's consider a hypothetical situation where the price of bacon goes up and the government offers to compensate Homer for that purchasing power loss. E.g., let's assume that Mayor Quimby realizes that more expensive bacon makes Homer really sad, and in an effort to keep Homer's spirits high (since Homer is a registered voter), the mayor offers to mail Homer a check that compensates him for this loss of purchasing power. In terms of an amount, one option would involve the mayor mailing a check that shifts Homer's new budget constraint BC_2 until that budget constraint is tangent to IC_1 (note that this check is the equivalent of a change in income, so this is a parallel shift in the budget constraint). That would allow Homer to return to his original level of utility, even though he'd consume a different amount of bacon than he did at X_1. Another option would be to mail a check that allows Homer to consume at X_1 again. This option would shift BC2 until that line intersects X_1. Let's assume the mayor chooses the first option.

Here is what Homer's indifference curve graph would look like after he receives the check.



If Homer is able to return to his original level of utility along IC_1, we assume that he is indifferent between buying bacon and all other goods at W, but facing the new prices, and buying bacon and all other goods at X_1, but facing the original prices.

Point W reflects the quantities of bacon and all other goods that Homer would buy after receiving his government check. Of course, in real life (an odd phrase to use in an example involving a cartoon character, but let's roll with it), mayors don't send checks to citizens who are disgruntled over bacon prices. Let's use the events leading to Homer supposedly consuming at point W, however, to make a couple observations.

First, what is the difference between W and X_1? There is no difference in utility since both points are located on IC_1, but the price ratio associated with BC_3 reflects a higher relative price for bacon. What these two points show us is the purchase Homer would make if he had no loss in purchasing power, but still had to face new prices. We observe how he would still make a purchasing substitution though.

Second, what is the difference between W and X_2? The prices that Homer faces at each point are clearly the same since BC_2 and BC_3 are parallel and the slope of each budget constraint is the ratio of those prices. What separates these two budget constraints is the equivalent of a change in income. Since income clearly did not change, perhaps we could alternatively characterize this as a change in purchasing power.

What do these observations teach us? The difference between W and X_1 stems from us looking at the effect of a changing bacon price while holding utility constant. By definition, this is called the Substitution effect. The difference between W and X_2 stems from us looking at the effect of changing purchasing power while holding prices constant at our new price ratio. By definition, this is called the Income effect.

What would Homer consume at pt. W? The calculation is somewhat involved, so we'll skip the details, but at least point out that we must satisfy 3 conditions (below).

$$10Q_A + 4Q_B = 100 + \Delta I \qquad \text{(a)}$$

$$-\frac{Q_A}{Q_B} = -\frac{4}{10} \qquad \text{(b)}$$

$$\sqrt{Q_B \cdot Q_A} = 5\sqrt{5} \qquad \text{(c)}$$

Condition (a) is Homer's budget constraint with the new prices ($P_A = 10$, $P_B = 4$) and the extra income represented by the mayor's check. Condition (b) states that point W must involve a tangency point between IC_1 and BC_3. Our final condition (c) reminds us that when consuming at point W, Homer must achieve his original level of utility (which we calculated earlier to be $5\sqrt{5}$). When we solve for DI, we discover that Homer's check will be in the amount of $41.42. With this information, we can determine that at point W, $Q_A^* = 7.1$ and $Q_B^* = 17.7$.

The overall effect of this price change is that Homer's consumption of bacon is lowered from 25 units to 12.5 units. That overall effect consists of the substitution effect and income effect. The substitution effect associated with bacon in this example is the difference between Q_B^* at points X_1 and W, which is 7.3 units of bacon. The income effect is the difference between Q_B^* at points X_2 and W, which is 5.2 units of bacon.