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Economics 301

Indifference Curves and the Consumer Equilibrium

Let’s assume that a representative consumer named Homer Simpson consumes beer and pork rinds in varying amounts. Assume further that the overall utility he derives from consuming these goods can be described by the utility function below. Note that this is just one possible example of a utility function, that there are many other possible functions we could have used instead.

(1) $$U = \sqrt{Q_B \cdot Q_R}$$

We can use this utility function to derive Homer's indifference curve. By setting (1) equal to a specific number, we are saying that there are various combinations of Q_B and Q_R that yield a level of utility equal to that specific number. For example, suppose we set Homer's utility function equal to 100. We derive the indifference curve allowing 100 units of utility (i.e. utils) by rearranging the equation as follows.

(1a) $$100 = \sqrt{Q_B \cdot Q_R}$$

If we square both sides of (1a), we get:

(1b) $$10{,}000 = Q_B \cdot Q_R$$

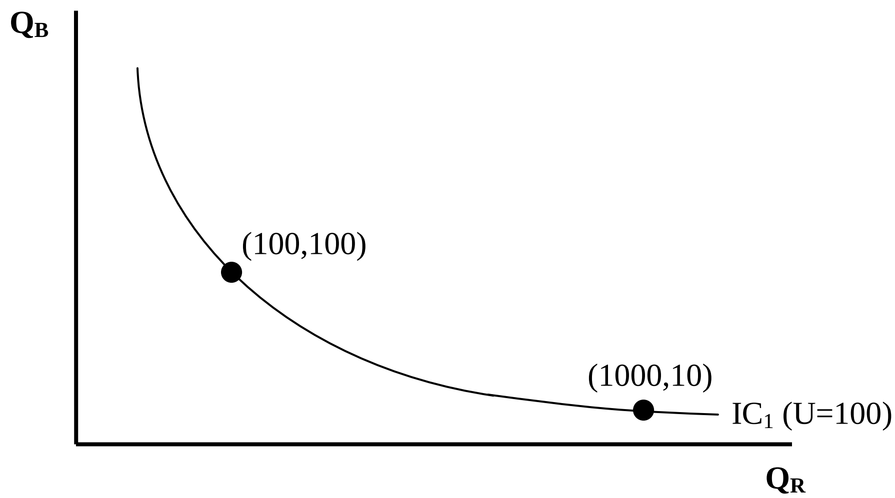
And then solving (1b) for Q_B gives us an equation that represents an indifference curve:

(1c) $$Q_B = \frac{10{,}000}{Q_R}$$

As stated above, (1c) tells us the various combinations of beer and pork rinds that will provide Homer with 100 utils of satisfaction.

For example, if Homer consumes 10 units of beer, he needs to consume 1,000 units of pork rinds to get 100 utils of satisfaction. Of course, this equation also tells us that Homer would be indifferent between consuming that bundle of goods (10 units of beer and 1,000 units of pork rinds) and another one with 100 units of beer and 100 units of pork rinds. This is because both bundles provide 100 utils of satisfaction.

The graph that goes with (1c) is pictured below. The two different consumption points we just discussed are pictured too (with their coordinates reported as (Q_R,Q_B). Both are on the indifference curve, both yield 100 utils of satisfaction.



Not knowing whether Homer will actually consume at either of these points, or whether he'll even consume on this indifference curve, we turn now to figuring out where Homer's consumption will actually occur. To do this we need a couple pieces of missing information: (a) the slope of the indifference curve, and (b) the budget constraint equation.

In a model where we examine two goods simultaneously, the slope of the indifference curve is going to be the marginal utility related to consuming more of one good divided by the marginal utility related to consuming less of the other good. While the utility along any indifference curve is constant, the marginal utility is not.

The marginal utility (MU) for each good above is given as:

$$MU_B = \frac{Q_R}{2\sqrt{Q_B Q_R}} \qquad \text{where } MU_B = \frac{\partial U}{\partial Q_B}$$

$$MU_R = \frac{Q_B}{2\sqrt{Q_B Q_R}} \qquad \text{where } MU_R = \frac{\partial U}{\partial Q_R}$$

The slope of the indifference curve, called the marginal rate of substitution, will be $-(MU_R/MU_B)$

Note that the slope of this curve is negative (to see this mathematically, consider (1c)), which means we write the marginal rate of substitution for pork rinds and beer (MRS_{RB}) as:

(2) $$MRS_{RB} = -\frac{Q_B}{Q_R}$$

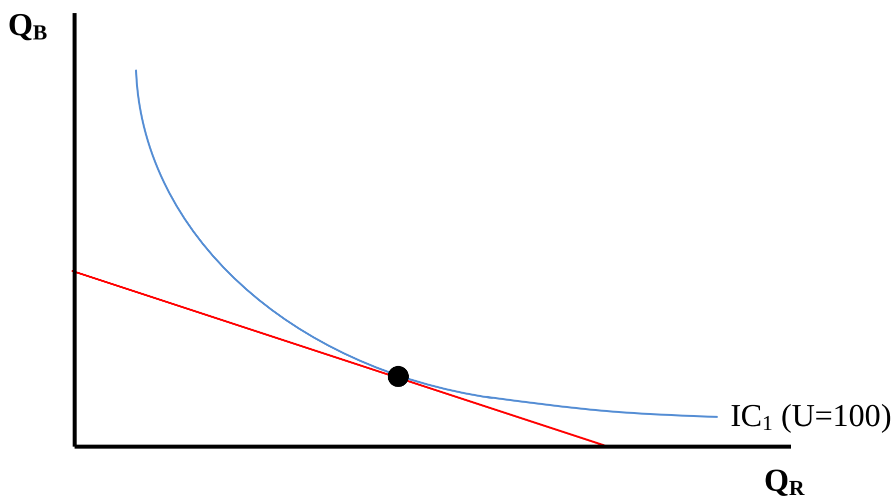
We'll assume that the price of beer is $4 and that the price of pork rinds is $2. Assume further that Homer's income is $200. The budget constraint is then given as:

(3) $4Q_B + 2Q_R = 200$

Rearranging (3), by solving for Q_B, we get the following (rearranged budget constraint):

(3a) $Q_B = -0.5\, Q_R + 50$

Noting that (3a) is the equation of a line (slope of –0.5, vertical intercept of 50), we can graph the indifference curve and budget constraint together. Equilibrium is attained where the (blue) indifference curve is tangent to the (red) budget constraint. This point is included in the graph.



The graph enables us to visually determine equilibrium, but also note the two conditions which must simultaneously occur when we are at this equilibrium point. Those conditions are:

- the slope of the two curves must be equal (i.e. $MRS_{R,B} = -P_R/P_B$)
- our consumer must be on their budget constraint (i.e. $Q_B + 2Q_R = 200$)

With this in mind, we can now solve for equilibrium here. Substitute the values of the slopes into the first condition.

(4) $$-\frac{Q_B}{Q_R} = -0.5$$

Solve (4) for Q_B

(4a) $Q_B = 0.5Q_R$

Substitute (4a) into the budget constraint (for Q_B).

(5) $4(0.5Q_R) + 2Q_R = 200$

Solve (5) for Q_R. This is the equilibrium value for Q_R (i.e. Q_R*).

$Q_R^* = 50$

Plug Q_R* into the original budget constraint (or (4)), and solve for Q_B. This is the equilibrium value for Q_B (i.e. Q_B*).

$4Q_B + 2(50) = 200$
$Q_B^* = 25$

Given Homer's budget constraint and utility function, Homer should consume 25 units of beer and 50 units of pork rinds. If he does this, then his overall utility will be:

$\sqrt{25 \cdot 50} = 25\sqrt{2}$ which is about 35.4

Homer will experience about 35.4 utils of satisfaction from buying 25 units of beer and 50 units of pork rinds.