Dr. Barry Haworth
University of Louisville
Department of Economics
Economics 301

Equilibrium in the Indifference Curve model

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1. **Indifference curves**. Let's begin by assuming we have a very typical consumer who purchases 2 different goods, we'll call them good 1 and good 2, but you can think of this as candy bars and milk, or some specific good like cereal and a composite good we'll call "all other goods". Whatever we call these goods, we know that the consumer derives happiness from purchasing the goods. In other words, as this consumer buys more good 1 or good 2 (ceteris paribus), our consumer is happier.

Does this happiness continue forever? No, obviously not. E.g., Cocoa Puffs is the greatest cereal ever invented, but within a given period, there's a point where consuming more of that otherwise awesome cereal would not make me happier. This would be the point where I become a Cocoa Puffs volcano, not a pretty sight. It's probably safe to assume, however, that I'll probably never get to that point because it'd take a lot of Cocoa Puffs to put me in that situation. Therefore, we can assume that every time I consume one of those awesome little chocolatey spheres, each sphere makes me happier. Does my happiness increase by the same amount every time? No, at some point, each additional chocolatey sphere might increase my overall happiness, but not by as much as the last one. That is, we also know that my happiness with goods like Cocoa Puffs increases at a decreasing rate. I.e., my happiness goes up with every bite, but that rate of increase starts slowing down and I'm not nearly as excited to eat the last few bites as I was when eating the first few.

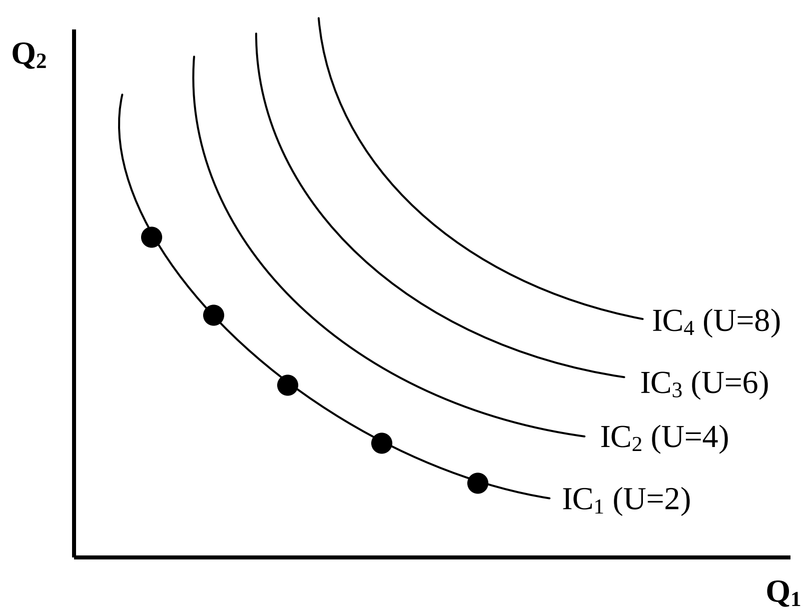
We've already assumed that more of good 1 or good 2 is better than less. Let's also assume that when our consumer purchases goods 1 and 2, the consumer places those goods in a basket (e.g. think of this as one of those baskets you can use at the grocery store). Looking at each of the potential combinations of these goods that our consumer could place in her basket, we'll assume that our consumer can rank those different possible baskets - in terms of whether she prefers one basket over another, or is indifferent between the baskets. We'll further assume that our consumer will make choices involving these different possible baskets in a manner where those choices are consistent with one another. E.g., if she prefers basket A to basket B, and basket B to basket C, then it should also be true that she prefers A to C as well. If not, then we'd have problems trying to predict her behavior.

All of that said, we know that there are likely going to be a bunch of potential baskets that make her equally happy. This is similar to when one of your friends asks whether you want to go to one movie or another one, and you really don't care which movie you watch, but we'll assume that you have more than just 2 choices (i.e. baskets) which provide you with a certain amount of happiness. For the sake of this example, let's say that happiness is measured as a level of what we'll call utility, and that utility can be expressed as a function (e.g. $U = f(Q_1,Q_2)$, where Q_1 is the quantity of good 1 you consume, and Q_2 is the quantity of good 2 you consume, and U is the level of utility you get from each possible purchase or consumption of these goods). Like we said before, as you plug greater quantities of good 1 or good 2 into our function, the utility you derive from those purchases goes up (i.e. U increases).

Let's assume that the following possible purchases of goods 1 and 2 yield the same amount of utility. If you'd like, you can verify that our function here would be $U = \sqrt{Q_1 \cdot Q_2}$

Q_1	Q_2	U
1	4	2
4/3	3	2
2	2	2
3	4/3	2
4	1	2

This tables tells us that our consumer would be equally happy with receiving 1 unit of good 1 and 4 units of good 2 as she'd be in receiving 4 units of good 1 and 1 unit of good 2. If we put all of these different combinations for goods 1 and 2 on a graph with Q_2 on the vertical axis and Q_1 on the horizontal axis (note that it doesn't matter which good goes on which axis), then connect the dots, we'd basically get something that looks like the curve IC_1 on the graph below (where U = 2 is what we get from each of those combinations on IC_1).



We call IC_1 an indifference curve, because our consumer is indifferent between each of the combinations of goods 1 and 2 on that curve, because each combination gives her U = 2. The other IC curves represent other possible combinations that would (as a group) provide our consumer with still higher amounts of utility.

2. **The Budget Constraint.** What keeps our consumer from jumping out to indifference curve IC_4? Although that curve makes her more happy than IC_1, IC_2 or IC_3, she doesn't get these combinations for free, she must buy them and her income may not allow her to achieve the level of happiness (utility) associated with IC_4.

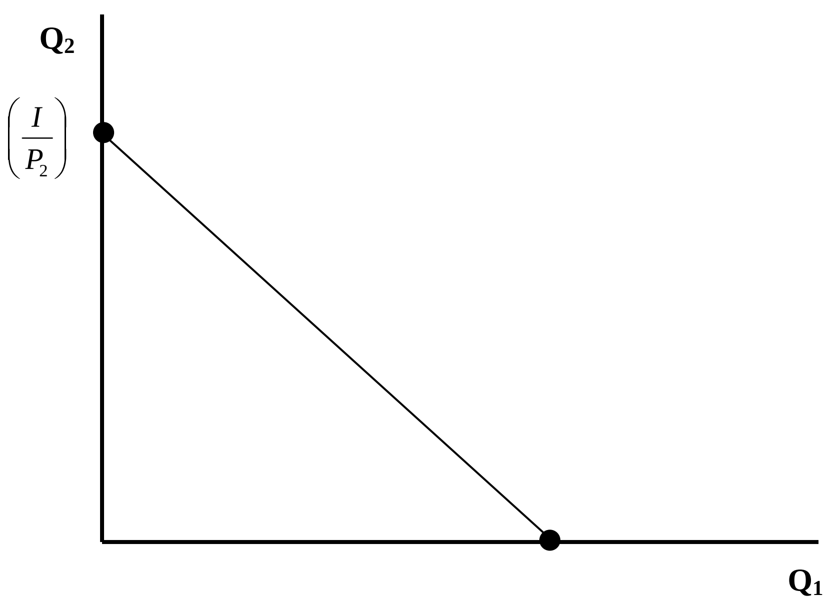
Therefore, we must also account for how our consumer's income places a constraint on her purchasing decision. Let's assume that our consumer spends all of her income (i.e. no savings), which means that the sum of her expenditure on goods 1 and 2 is equal to her income. That thought can be expressed in terms of a budget constraint.

Her Budget Constraint is: $I = P_1Q_1 + P_2Q_2$

In this equation, I = income, P = price of goods 1 and 2, Q = quantity of goods 1 and 2 our consumer chooses to buy. The I and P variables are set, i.e. constants, in that we will ultimately substitute a numerical value for income and price (e.g. we could assume that I = $20,000 and that P_1 and P_2 are both equal to $2). Given that the purchase of this equation will be to ultimately determine the amount of goods 1 and 2 that a consumer wants to buy, the Qs will remain as variables here. I.e. if I = $20,000 and P_1 and P_2 are both equal to $2, then (dropping the $ signs to keep things simple) the equation would ultimately be written as $20,000 = 2Q_1 + 2Q_2$. If we rearrange her Budget Constraint equation into something we can put on a graph, like this:

$$Q_2 = -\left(\frac{P_1}{P_2}\right)Q_1 + \left(\frac{I}{P_2}\right)$$

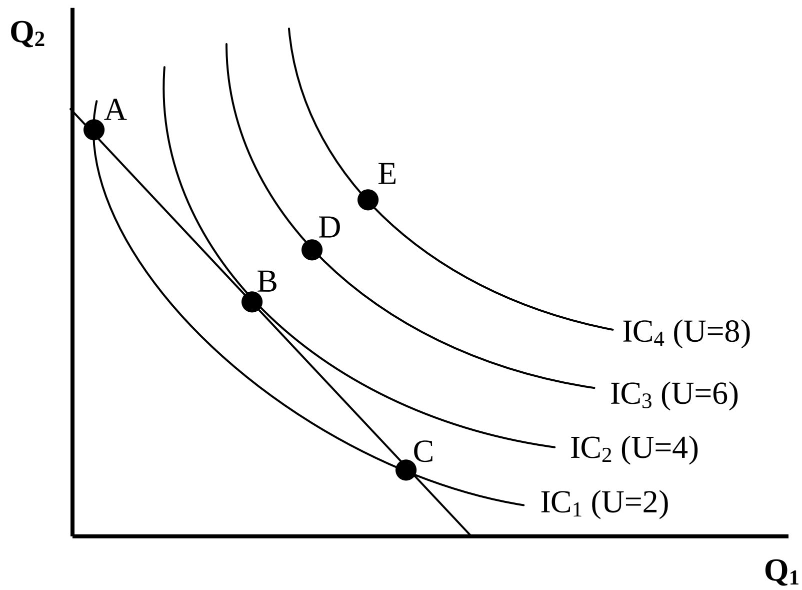
Notice that since her income and these prices are going to be (constant) numerical values, this equation is giving us the same information as a more traditional equation from your past. I.e., the equation for a line with the variables x and y, $y = mx + b$. If we put the rearranged budget constraint equation on a graph where Q2 is on the vertical axis and Q1 on the horizontal axis, then we have this (where the slope of the line is $-P_1/P_2$ and the vertical intercept is I/P_2).



Intuitively, we may recognize that the ratio of prices (P_1/P_2) represents a comparison of the cost to consumers of one unit of each good. Therefore, in a sense, we can say that P_1/P_2 is the ratio of the marginal cost of goods 1 and 2 respectively. Recalling our macroeconomic discussion of price indexes, we see that I/P_2 is a measure of purchasing power in terms of good 2. If P_2 falls, I/P_2 gets bigger - which means that our purchasing power has increased.

3. **Where does equilibrium occur?** We know that our consumer will choose to spend her money in such a way that she can achieve the greatest possible amount of happiness (utility). On our graph, that means while she's seeking to locate on the highest possible indifference curve, she must also operate within the constraints of her budget.

As long as she remains on her Budget Constraint line, she is spending all of her income. Of course, as we've been saying, her income can only allow her to purchase just so much. That means that the point where she locates must be somewhere on her Budget Constraint line. E.g., she could locate on the graph (below) at points A, B or C, but not at points D or E.



If she locates at points A or C, then our graph tells us she'd get U = 2 from either basket or combination. That's good, but not quite as good as what she'd get a point B. At point B, she would receive U = 4, which is better than U = 2. Given that she must pick a combination that's represented by a point on her Budget Constraint line and will want to choose one that gives her the greatest possible amount of happiness (utility), she'll obviously make a purchase that involves the intersection of her Budget Constraint line with the highest possible IC curve. This occurs at point B, a point that involves tangency between her Budget Constraint and IC_2.

Tangency points like point B are points where two curves obviously intersect, but also where the slope of each curve is equal. We know from our discussion above that the slope of the budget constraint in this example is $-(P_1/P_2)$, but we haven't said anything about the slope of these indifference curves. Since these curves are not straight lines, the slope of the curve changes as you move from point to point. Therefore, we must refer to the slope of the indifference curve in terms of a specific point. Rather than derive the slope mathematically, we'll simply call it by its name, the marginal rate of substitution, and abbreviate the slope of the indifference curve as MRS. Our equilibrium can then also be expressed as occurring where MRS = $-(P_1/P_2)$.

We can lastly state that our equilibrium in this example will occur where 2 conditions are met.

(1) We are operating on the budget constraint, i.e. it must be true that $I = P_1Q_1 + P_2Q_2$

(2) Our equilibrium will involve a tangency point between the budget constraint and our eventual indifference curve, i.e. it must be true that MRS = $-(P_1/P_2)$