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Economics 201

The relationship between own-price elasticity and total revenue

If the price of a good increases, then total revenue is affected. E.g., if the price of a good or service increases, then we know that this higher price should lead to a lower quantity sold, but because higher prices and lower quantities sold affect revenue differently (ceteris paribus), does that price change increase or decrease revenue? This handout looks at that relationship.

We begin by assuming a demand curve for what we'll call the market for good X. That demand curve is given below.

$P_X = 500 - 2Q_X$ (where P_X = price of good X, Q_X = quantity demanded of good X)

We also know that an own-price elasticity can be calculated at each point on this demand curve. The formula for own-price elasticity is also given below.

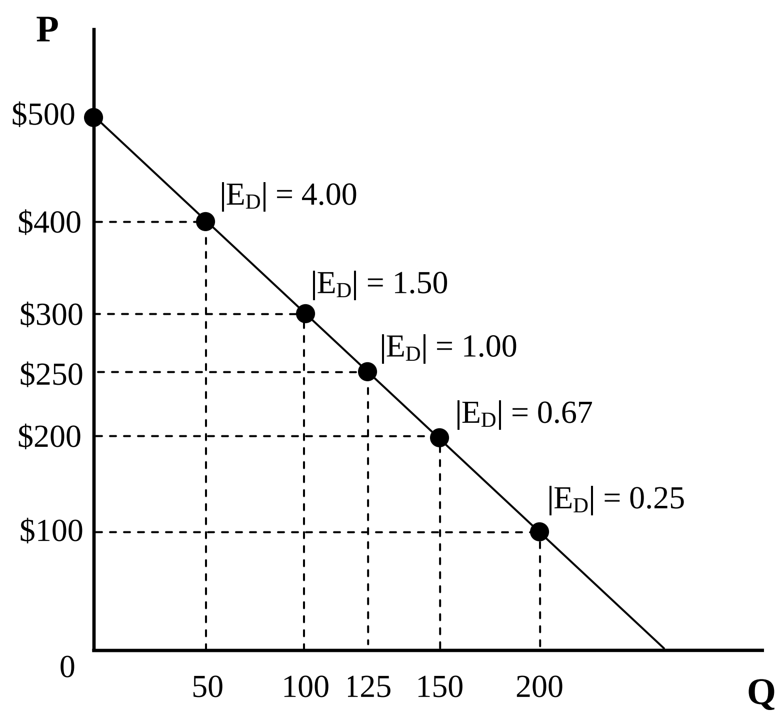
$$E_D = \frac{\%\Delta Q_X}{\%\Delta P_X}$$

Given that $\%\Delta Q_X = \frac{\Delta Q_X}{Q_X}$ and $\%\Delta P_X = \frac{\Delta P_X}{P_X}$, we know that we can manipulate this own-price elasticity equation from its current form into this: $E_D = \left(\frac{\Delta Q_X}{\Delta P_X}\right)\left(\frac{P_X}{Q_X}\right)$. Of course, we may want to ask, what does this equation tell us? Note that the change in Q_X divided by the change in P_X is simply the inverse of the slope of this demand curve (run over rise, rather than rise over run). Given the demand curve equation above, we know that $\frac{\Delta Q_X}{\Delta P_X} = \frac{1}{-2}$. The values inside the other parenthesis are something we can obtain by plugging values for Q_X into the demand curve equation, giving us corresponding values for P_X. For example, when we plug $Q_X = 100$ into the demand curve equation, we get $P_X = 300$. That would give us an own-price elasticity calculation of $\left(\frac{1}{-2}\right)\left(\frac{300}{100}\right)$, or -1.5, at this specific point on the demand curve (i.e. the point where $Q_X = 100$ and $P_X = 300$).

We can do the same thing for several other points on the demand curve. In each case, we consider a different value for Q_X, and then calculate corresponding value for P_X. If we plug each pair of values into our revised equation for own-price elasticity (i.e. $E_D = \left(\frac{\Delta Q_X}{\Delta P_X}\right)\left(\frac{P_X}{Q_X}\right)$), we can calculate an own-price elasticity measure that corresponds with that pair of values. The table below does this for several possible values of P_X, along with an interpretation of our own-price elasticity values (in terms of whether those values are elastic or inelastic).

Q_X	P_X	$E_D = \left(\frac{1}{-2}\right)\left(\frac{P_X}{Q_X}\right)$	Interpretation
50	400	-4.00	elastic
100	300	-1.50	elastic
125	250	-1.00	unit elastic
150	200	-0.67	inelastic
200	100	-0.25	inelastic

On a graph, that gives us the following (note that we report the elasticity calculations in terms of absolute value):



Note that the own-price elasticity of each point increases as we move up the demand curve. As it turns out, this is true for every linear demand curve.

Although we're close to answering our initial question, we still haven't answered it yet. That said, how does a change in price affect total revenue in this setting? Let's answer that question by considering how total revenue would change at each of these points if the price was to rise by

$2, moving from $1 below each of the given prices from above and increasing to $1 above each price. We'll then do the same analysis with the price at each point above falling by $2. In the first table below, we calculate the total revenue associated with each of the points on our graph and then again in the case where the price rises by $2.

Change in TR if Price Increases (PX_1 to PX_2)

Q_{X1}	P_{X1}	TR_1	Q_{X2}	P_{X2}	TR_2	ΔTR?
50.5	399	20,149.5	49.5	401	19,849.5	-
100.5	299	30,049.5	99.5	301	29,949.5	-
125.5	249	31,249.5	124.5	251	31,249.5	no Δ
150.5	199	29,949.5	149.5	201	30,049.5	+
200.5	99	19,849.5	199.5	101	20,149.5	+

Note that when we move across the points that are elastic, we observe price increases leading to a fall in total revenue. When we are moving across the inelastic points, total revenue increases, and when we move across the point that is unit elastic, total revenue doesn't change (which would imply a maximum point).

Change in TR if Price Decreases (PX_1 to PX_2)

Q_{X1}	P_{X1}	TR_1	Q_{X2}	P_{X2}	TR_2	ΔTR?
49.5	401	19,849.5	50.5	399	20,149.5	+
99.5	301	29,949.5	100.5	299	30,049.5	+
124.5	251	31,249.5	125.5	249	31,249.5	no Δ
149.5	201	30,049.5	150.5	199	29,949.5	-
199.5	101	20,149.5	200.5	99	19,849.5	-

When price decreases, we clearly get just the opposite result – price decreases lead to an increase in total revenue when moving across points that are elastic, and price decreases lead to a decrease in total revenue when moving across inelastic points.

This is not particularly hard to verify. If we increase the price of a good, then (ceteris paribus) that has a positive effect on total revenue. Of course, a higher price leads to fewer units being sold, and that decrease in quantity (ceteris paribus) has a negative effect on total revenue. When the positive effect is larger than the negative effect, we expect total revenue to increase. This would be occur when moving along the elastic segment of the demand curve. In contrast, we would get the opposite result when moving along the inelastic segment of the demand curve.