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Economics 201

**The relationship between own-price elasticity and total revenue**

If the price of a good increases, then total revenue is affected. E.g., if the price of a good or service increases, then we know that this higher price should lead to a lower quantity sold, but because higher prices and lower quantities sold affect revenue differently (ceteris paribus), does that price change increase or decrease revenue? This handout looks at that relationship.

We begin by assuming a demand curve for what we'll call the market for good X. That demand curve is given below.

$P_X = 500 - 2Q_X$ (where $P_X$ = price of good X, $Q_X$ = quantity demanded of good X)

We also know that an own-price elasticity can be calculated at each point on this demand curve. The formula for own-price elasticity is also given below.

$$E_D = \frac{\%\Delta Q_X}{\%\Delta P_X}$$

Given how we calculate a percentage change, we know that $\%\Delta Q_X = \frac{\Delta Q_X}{Q_X}$ and $\%\Delta P_X = \frac{\Delta P_X}{P_X}$, which implies that we can manipulate this own-price elasticity equation from its current form into equation (1) below

(1) $$E_D = \left(\frac{\Delta Q_X}{\Delta P_X}\right)\left(\frac{P_X}{Q_X}\right)$$

Whether it's clear how we got to this point or not, let's take this equation as given and interpret it. Consider the first parenthetic part of equation (1), i.e. $\frac{\Delta Q_X}{\Delta P_X}$. Note that this expression is simply the inverse of the slope of this demand curve (run over rise, rather than rise over run).

If we work with the demand equation above, then we know that $\frac{\Delta Q_X}{\Delta P_X} = \frac{1}{-2}$. If we plug this into equation (1) above, then we have:

(1) $E_D = \left(\frac{1}{-2}\right)\left(\frac{P_X}{Q_X}\right)$

Since the slope of this demand curve is constant, then we know that whenever we work with the demand curve above (or any other demand curve with a slope of -2, the equation above won't change.

Now, let's consider the second parenthetic part of this equation. This is simply Price divided by Quantity, and if we plug values for $Q_X$ into the demand curve equation, we'll get corresponding values for $P_X$. E.g., when we plug $Q_X$ = 100 into the demand curve equation, we get $P_X$ = 300. Plugging $Q_X$ = and $P_X$ = 300 into equation (1) above, we get:

(1) $E_D = \left(\frac{1}{-2}\right)\left(\frac{300}{100}\right)$

I.e., $E_D$ = -1.5. In other words, if we draw a graph of the demand curve from this example, and locate the point where $Q_X$ = 100 and $P_X$ = 300, we realize that this point has an elasticity where $E_D$ = -1.5.

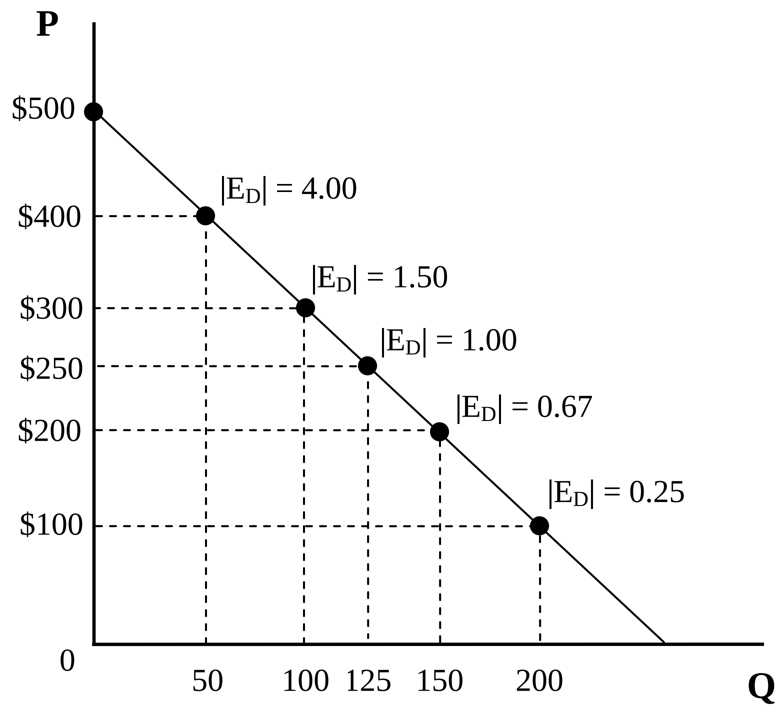
We can do the same thing for several other points on the demand curve. In each case, we consider a different value for $Q_X$ , and then calculate corresponding value for $P_X$ . If we plug each pair of values into our revised equation (1), then we can calculate an own-price elasticity value for other points on the demand curve.

The table below does this for several possible values of $P_X$, along with an interpretation of our own-price elasticity values (in terms of whether those values are elastic or inelastic).

| **$Q_X$** | **$P_X$** | $E_D = \left(\frac{1}{-2}\right)\left(\frac{P_X}{Q_X}\right)$ | **Interpretation** |
|---|---|---|---|
| 50 | 400 | -4.00 | elastic |
| 100 | 300 | -1.50 | elastic |
| 125 | 250 | -1.00 | unit elastic |
| 150 | 200 | -0.67 | inelastic |
| 200 | 100 | -0.25 | inelastic |

Let's take the values from the first 3 columns of this table and plot everything on a graph of the demand curve we're working with in this problem.

Here's how that graph would appear.




Note that the own-price elasticity of each point increases as we move up the demand curve. I.e., the points get more elastic. As it turns out, this is true for every linear demand curve.

Although we're close to answering our initial question, we still haven't answered it yet. That said, how does a change in price affect total revenue in this setting? Let's answer that question by considering how total revenue would change at each of these points if the price was to rise by $2, moving from $1 below each of the given prices from above and increasing to $1 above each price. We'll then do the same analysis with the price at each point above falling by $2. In the first table below, we calculate the total revenue associated with each of the points on our graph and then again in the case where the price rises by $2.

**Change in TR if Price Increases ($PX_1$ to $PX_2$)**

| $Q_{X1}$ | $P_{X1}$ | $TR_1$ | $Q_{X2}$ | $P_{X2}$ | $TR_2$ | $\Delta TR$? |
|---|---|---|---|---|---|---|
| 50.5 | 399 | 20,149.5 | 49.5 | 401 | 19,849.5 | - |
| 100.5 | 299 | 30,049.5 | 99.5 | 301 | 29,949.5 | - |
| 125.5 | 249 | 31,249.5 | 124.5 | 251 | 31,249.5 | no $\Delta$ |
| 150.5 | 199 | 29,949.5 | 149.5 | 201 | 30,049.5 | + |
| 200.5 | 99 | 19,849.5 | 199.5 | 101 | 20,149.5 | + |

Note that when we move across the points that are elastic, we observe price increases leading to a fall in total revenue. When we are moving across the inelastic points, total revenue increases, and when we move across the point that is unit elastic, total revenue doesn't change (which would imply a maximum point).

**Change in TR if Price Decreases ($PX_1$ to $PX_2$)**

| $Q_{X1}$ | $P_{X1}$ | $TR_1$ | $Q_{X2}$ | $P_{X2}$ | $TR_2$ | $\Delta$TR? |
|---|---|---|---|---|---|---|
| 49.5 | 401 | 19,849.5 | 50.5 | 399 | 20,149.5 | + |
| 99.5 | 301 | 29,949.5 | 100.5 | 299 | 30,049.5 | + |
| 124.5 | 251 | 31,249.5 | 125.5 | 249 | 31,249.5 | no $\Delta$ |
| 149.5 | 201 | 30,049.5 | 150.5 | 199 | 29,949.5 | - |
| 199.5 | 101 | 20,149.5 | 200.5 | 99 | 19,849.5 | - |

When price decreases, we clearly get just the opposite result – price decreases lead to an increase in total revenue when moving across points that are elastic, and price decreases lead to a decrease in total revenue when moving across inelastic points.

This is not particularly hard to verify. If we increase the price of a good, then (ceteris paribus) that has a positive effect on total revenue. Of course, a higher price leads to fewer units being sold, and that decrease in quantity (ceteris paribus) has a negative effect on total revenue. When the positive effect is larger than the negative effect, we expect total revenue to increase. This would be occur when moving along the elastic segment of the demand curve. In contrast, we would get the opposite result when moving along the inelastic segment of the demand curve.

Summarizing, here is our result:

- If demand is elastic and P↑, then total revenue decreases
- If demand is elastic and P↓, then total revenue increases
- If demand is inelastic and P↑, then total revenue increases
- If demand is inelastic and P↓, then total revenue decreases