Dr. Barry Haworth
University of Louisville
Department of Economics
Economics 201

Product and Cost Curves

The point of this handout is to illustrate some basic points about product and cost curves. Below, we will address various ways of measuring output and learning how factors contribute to producing that output. When we measure the cost of production, we'll want to make some similar calculations there.

Product Curves

Consider Table 1 (next page). The table assumes that a firm is producing a specific good using varying amounts of labor (L) with a fixed amount of capital (K). To keep things simple, we'll assume that labor is individual laborers (people) and that capital is the number of machines used in production.

Assume that the production function for this firm is: $Q = 10 + 10L + 20L^2 - 0.6L^3$. Hiring more labor increases output (Q), but as the table demonstrates, output is increasing at a decreasing rate, because although output is increasing, the change in output starts getting smaller after the 12th unit of labor is hired. After hiring the 22nd laborer, we note that output actually begins decreasing as well. Quite obviously, no firm would ever hire labor at the point where doing so decreases output, and so we know that the most laborers that this firm would ever hire is 22.
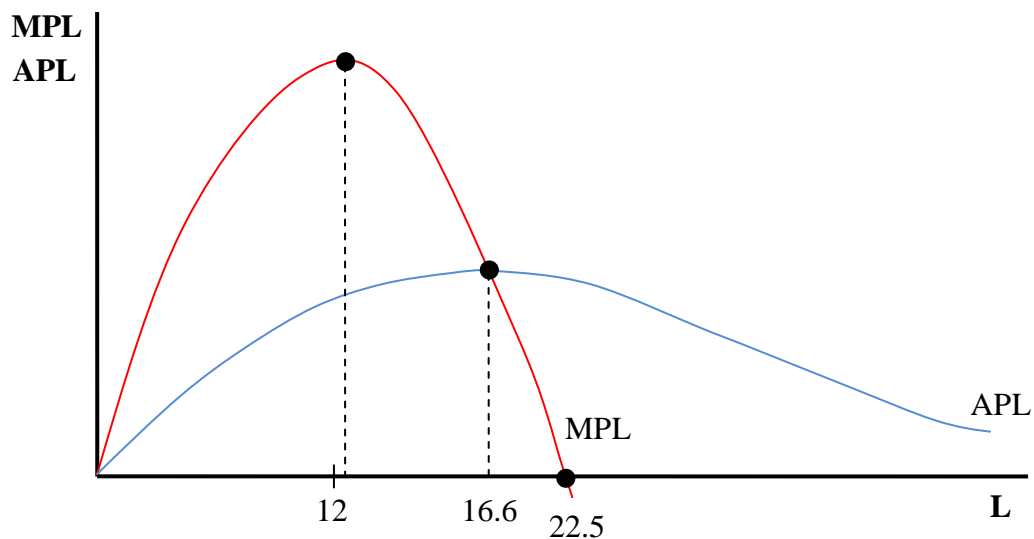
Once we know the output, we can determine how our variable factor contributes to the production of that output. One measure involves calculating the number of units of output that each laborer is responsible for producing on average. We call this the average product of labor (APL). The APL is calculated as Q/L. That value is calculated in column 4 of table 1. We see that APL increases until we hire between 16 and 17 laborers, and then decreases. APL continues to decrease, but never becomes negative (which makes sense if you only hire positive amounts of labor and can't produce a negative number of units).

Another way to view the effect of hiring more labor is to consider the impact of each unit of labor hired on output. We call this the marginal product of labor (MPL), and it's calculated as $\Delta Q / \Delta L$. Since we increase labor by 1 unit at a time, we can calculate MPL in this example as simply ΔQ. That value is presented in column 5 of table 1. We note that MPL also increases to a certain point (L = 12) and then begins decreasing, eventually becoming negative. Note that MPL becomes negative at the point where we observe output begin decreasing (after L = 22).

Table 1

L	K	Q	APL	MPL
0	4			
1	4	39.4	39.4	39.4
2	4	105.2	52.6	65.8
3	4	203.8	67.9	98.6
4	4	331.6	82.9	127.8
5	4	485.0	97.0	153.4
6	4	660.4	110.1	175.4
7	4	854.2	122.0	193.8
8	4	1062.8	132.9	208.6
9	4	1282.6	142.5	219.8
10	4	1510.0	151.0	227.4
11	4	1741.4	158.3	231.4
12	4	1973.2	164.4	231.8
13	4	2201.8	169.4	228.6
14	4	2423.6	173.1	221.8
15	4	2635.0	175.7	211.4
16	4	2832.4	177.0	197.4
17	4	3012.2	177.2	179.8
18	4	3170.8	176.2	158.6
19	4	3304.6	173.9	133.8
20	4	3410.0	170.5	105.4
21	4	3483.4	165.9	73.4
22	4	3521.2	160.1	37.8
23	4	3519.8	153.0	-1.4
24	4	3475.6	144.8	-44.2
25	4	3385.0	135.4	-90.6
26	4	3244.4	124.8	-140.6
27	4	3050.2	113.0	-194.2
28	4	2798.8	100.0	-251.4
29	4	2486.6	85.7	-312.2
30	4	2110.0	70.3	-376.6

If we take the information from table 1 and insert it into a graph, we have:

MPL
APL

12   16.6
      22.5
                              MPL

                                                    APL

                                                            L

The graph shows several points where the relative position of the average and marginal product curves and reinforces our comments above, where we discuss how the average product of labor and marginal produce of labor are changing as more labor is hired. The graph also illustrates the fact that MPL and APL intersect at the point where APL is at a maximum.

Cost Curves

Now we consider the cost associated with producing output. Note that when we discuss cost within the context of firms, we are discussing economic cost, rather than accounting cost. If we consider the cost of all factors, both variable and fixed factors, we get the overall cost of producing a given amount of output that we call total cost (TC). Let's assume that the total cost of producing varying amounts of output of some good is given by $TC = 2Q^3 - 18Q^2 + 60Q + 50$.

Table 2 shows us the total cost associated with each different output level from column 1. Note, of course, that higher output always leads to greater total cost.

One question we may want to ask is how much each unit of output costs the firm on average. This gives us a measure called average cost (AC), which we calculate as TC/Q. AC is provided in column 3 of our table. Note that AC decreases, and then increases (i.e. note that AC is a u-shaped curve in this example).
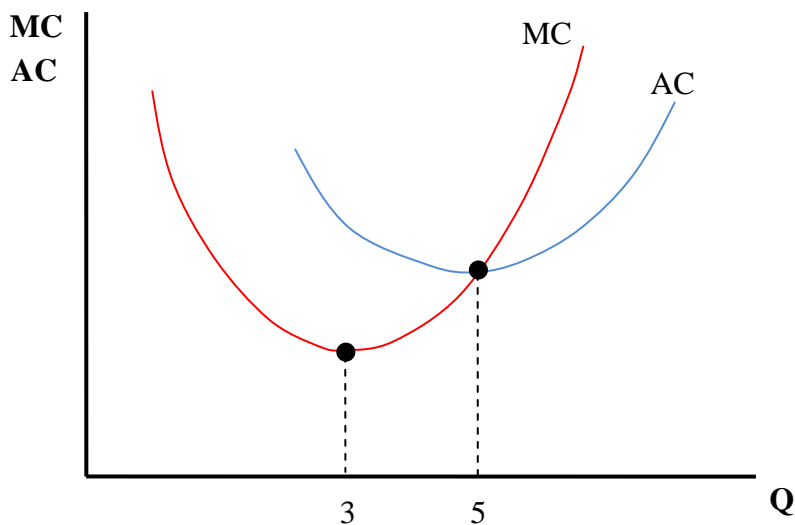
Table 2

Q	TC	AC	MC
1	94	94.0	30.0
2	114	57.0	12.0
3	122	40.7	6.0
4	130	32.5	12.0
5	150	30.0	30.0
6	194	32.3	60.0
7	274	39.1	102.0
8	402	50.3	156.0
9	590	65.6	222.0
10	850	85.0	300.0
11	1194	108.5	390.0
12	1634	136.2	492.0
13	2182	167.8	606.0
14	2850	203.6	732.0
15	3650	243.3	870.0
16	4594	287.1	1020.0
17	5694	334.9	1182.0
18	6962	386.8	1356.0
19	8410	442.6	1542.0
20	10050	502.5	1740.0
21	11894	566.4	1950.0
22	13954	634.3	2172.0
23	16242	706.2	2406.0
24	18770	782.1	2652.0
25	21550	862.0	2910.0
26	24594	945.9	3180.0
27	27914	1033.9	3462.0
28	31522	1125.8	3756.0
29	35430	1221.7	4062.0
30	39650	1321.7	4380.0

Another question about cost may relate to how cost changes as we produce more output. This leads us to marginal cost (MC). Marginal cost involves finding ΔTC/ΔQ, which we calculate in one of two ways. One method would involve doing what we did above to get MPL, divide the change in output by the change in labor hired. A more precise means of looking at MC, however, involves using calculus to come up with an equation for MC. Applying that approach to the TC equation above, we would get the following equation for MC: $MC = 6Q^2 - 36Q + 60$

How does total cost change as we produce each additional unit of output? Initially, MC will decrease just like AC as we produce additional output, but eventually, we observe MC increasing as output increases (again, just like AC). Therefore, the MC curve in this example is apparently also u-shaped.

As above, we can take the information from the table and draw the graph of average and marginal cost curves.

Both curves reach a minimum point. We identify te minimum point on the AC curve in the same manner we identified the minimum point on APL above. When AC and MC intersect (i.e. when AC = MC), we observe AC at a minimum point. This occurs when Q = 5.